Quantitative Reasoning on Hybrid Formulas with Dynamic Programming

Vu Phan's PhD thesis defense
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2022-07-14

- Motivation: probabilistic models quantify uncertainties in real-world applications
- Bridge: probabilistic models are reducible to Boolean formulas
- Statement: we can efficiently solve problems on Boolean formulas by partitioning

From Qualitative to Quantitative Reasoning

Boolean formula φ

- *SAT*: find a *satisfying assignment*, i.e., *model*, of φ [Coo71]
  - *Davis-Putnam-Logemann-Loveland (DPLL)* algorithm [DLL62]
  - *Conflict-driven clause learning (CDCL)* [MS96]

- *Weighted SAT*:
  - receive weights of assignments on vars (φ)
  - find a model of φ with the highest weight [SBK07]
- *Model counting*: find the number of satisfying assignments of φ [Val79]

Variable Eliminations

- Pseudo-Boolean function $f : \mathbb{B}^S \to \mathbb{R}$
- *Maximal projection* $\max_x f : \mathbb{B}^{S \setminus \{x\}} \to \mathbb{R}$

$$\left(\max_x f\right)(\tau) := \max\left(f\left(\tau \cup \{\langle x, 1\rangle\}\right), f\left(\tau \cup \{\langle x, 0\rangle\}\right)\right)$$

$$\left(\max_S f\right)(\varnothing) \in \mathbb{R}$$

- *Summative projection* $\sum_x f : \mathbb{B}^{S \setminus \{x\}} \to \mathbb{R}$

$$\left(\sum_x f\right)(\tau) := f\left(\tau \cup \{\langle x, 1\rangle\}\right) + f\left(\tau \cup \{\langle x, 0\rangle\}\right)$$

$$\left(\sum_S f\right)(\varnothing) \in \mathbb{R}$$

Quantitative Problems

- Boolean formula φ, where $\text{vars}(\varphi) = S$
- Boolean function $f = [\varphi] : \mathbb{B}^S \to \mathbb{B}$
- *Weight function* $W : \mathbb{B}^S \to \mathbb{R}_+$

Problem	Form	Notes	Complexity
Weighted SAT	$\max_S (f \cdot W)$	$\equiv$ *maximum SAT (MaxSAT), most probable explanation (MPE)*	NP-H
Model counting	$\sum_S f$	$\equiv$ probability of evidence, i.e., marginalization in Bayesian networks	#P-C
Projected counting	$\sum_X \max_Y f$	$\{X, Y\}$: partition of S	$\#\text{P}^{\text{NP}}$-C
Exist-random SAT (ERSAT)	$\max_X \sum_Y f$	$\equiv$ *maximum a posteriori (MAP)*	$\text{NP}^{\#\text{P}}$-H

Hybrid Constraints

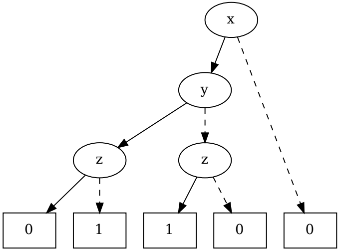
- *Conjunction normal form (CNF)* formulas are conjunctions of disjunctive clauses
- *Disjunctive clauses* (disjunctions of literals) alone can be inconvenient
  - *XOR clauses* (XORs of literals) are natural in cryptography [BKR11]
  - Performance of CNF encodings (e.g., [Tse83]) depends on solvers [Pre09]
- *XOR-CNF formulas* are conjunctions of disjunctive and XOR clauses

$$\varphi = x \wedge (x \vee \neg y) \wedge (y \oplus z) \quad \textit{factored representation}$$

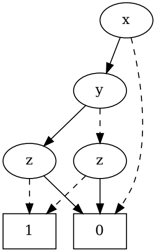
$$[\varphi] = [x] \cdot [x \vee \neg y] \cdot [y \oplus z] \quad \textit{(multiplicative) join}$$

Approaches to Solving Constraints

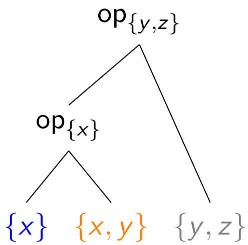
Model counting and SAT on XOR-CNF: $x \wedge (x \vee \neg y) \wedge (y \oplus z)$



Backtracking search: *binary decision tree*



Knowledge compilation: *binary decision diagrams (BDDs)* [Bry86], etc.



Dynamic programming: *project-join tree*, where

$$\mathrm{op} := \begin{cases} \sum & \text{model counting} \\ \max & \text{SAT} \end{cases}$$

Main Contribution: Versatile Framework for Quantitative Reasoning

- Dynamic programming: exploit factored representations [Pha19; DPV20a]
  - XOR-CNF formula: product of clauses
  - Assignment weight: product of literal weights
- Project-join tree T for an XOR-CNF formula φ: project out variables and conjoin clauses
  - *Planning phase*: build T from φ
  - *Execution phase*: traverse T to reason about φ

Single plan: multiple executions, one for each problem

Projection operators	Published (overlap with [Dud21, Jeff])	Archived (later submissions)
One: $\sum$, max	Model counting [DPV20b]	Weighted SAT [PV22b]
Two: $\sum$ max, max $\sum$	Projected counting [DPV21]	ERSAT [PV22a]

Progress

1. Model Counting
   - Planners
   - Executors
   - Evaluating Model Counters
2. Weighted SAT
   - Evaluating Weighted-SAT Solvers
3. Projected Counting
   - Evaluating Projected Counters
4. Exist-Random SAT (ERSAT)
   - Evaluating ERSAT Solvers

Model Counting

- Applications of model counting:
  - Analysis of information flows [KMM13]
  - Estimation of power reliability [Due+17]
- Input: a Boolean formula φ, where $S = \mathrm{vars}(\varphi)$
- Output: the number of assignments on S that satisfy φ

$$\#\varphi := \sum_{S} [\varphi] \qquad \textit{model count}$$

Early Projection: Push Variable Eliminations Inward

$$\sum_{x,y,z} f(x) \cdot g(x,y) \cdot h(y,z)$$

function over 3 variables: x, y, z

$$= \sum_{y,z} \left(\sum_{x} f(x) \cdot g(x,y) \right) \cdot h(y,z)$$

function over 2 variables: x, y

$$= \sum_{y,z} \left(\sum_{x} f(x) \cdot g(x,y) \right) \cdot h(y,z)$$

function over 1 variable: y

$$= \sum_{y,z} \left(\sum_{x} f(x) \cdot g(x,y) \right) \cdot h(y,z)$$

function over 2 variables: y, z

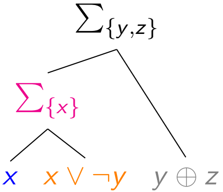
- Bayesian inference [ZP94]
- Database-query optimization [McM+04]

Project-Join Tree

XOR-CNF formula $\varphi = x \wedge (x \vee \neg y) \wedge (y \oplus z)$

Planning phase $\downarrow$

Project-join tree T



Execution phase $\downarrow$

$$f(x, y) = [x] \cdot [x \vee \neg y]$$

$$g(y) = \sum_{x} f(x, y)$$

$$h(y, z) = g(y) \cdot [y \oplus z]$$

$$\#\varphi = \sum_{y,z} h(y, z)$$

Model count $\#\varphi = 2$

Width of T $\quad \mathrm{width}(T) = 2$

Framework and Implementation

DPMC (dynamic-programming model counter) framework:

Planner: XOR-CNF formula $\mapsto$ project-join tree

Model counter (planner-executor pair)

Executor: project-join tree $\mapsto$ model count

Implementation:

Constraint-programming planner

Tree-decomposition planner

ADDMC [DPV20a]

Crossover1

Crossover2

TensorOrder [DDV19, Jeff]

Algebraic-decision-diagram executor

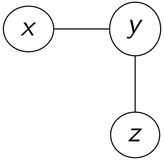
Tensor executor

Performance on single CPU cores:

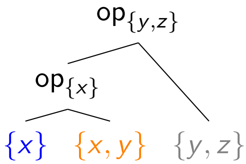
Crossover1 $>$ ADDMC $>$ TensorOrder $>$ Crossover2

Planning with Heuristics in *Constraint Programming (CP)*

XOR-CNF formula: $x \wedge (x \vee \neg y) \wedge (y \oplus z)$



Gaifman graph, i.e., *primal constraint graph*



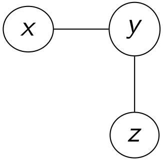
Project-join tree

CP heuristics:

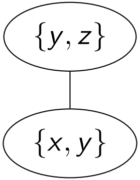
- Variable ordering:
  - *Maximum-cardinality search* [TY84]
  - *Minimum fill-in* [Dec03]
- Clause ordering:
  - *Bucket elimination* [Dec99]
  - *Bouquet's Method* [Bou99]

Planning with *Tree Decompositions (TDs)*

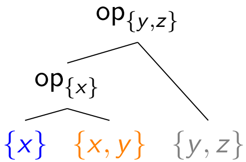
XOR-CNF formula: $x \wedge (x \vee \neg y) \wedge (y \oplus z)$



Gaifman graph



TD [RS91]



Project-join tree (Jeff)

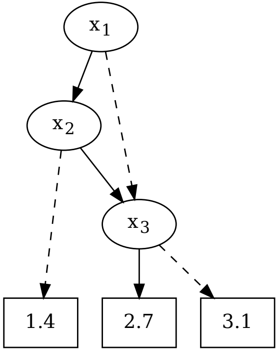
Tree decomposers from the PACE Challenge 2017 [Del+18]:

- `FlowCutter` [Str17]
- `Meiji` [Tam19]
- `htd` [AMW17]

Execution with Tensors and *Algebraic Decision Diagrams (ADDs)*

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
1	1	1	2.7
1	1	0	3.1
1	0	1	1.4
1	0	0	1.4
0	1	1	2.7
0	1	0	3.1
0	0	1	2.7
0	0	0	3.1

Tensor: *dense* representation (Jeff)

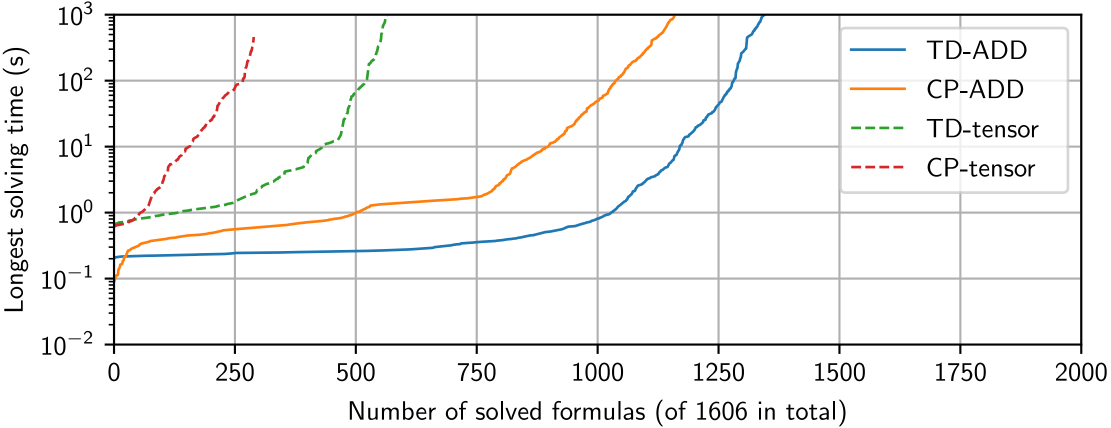


ADD: *sparse* representation

Evaluation of Model Counters

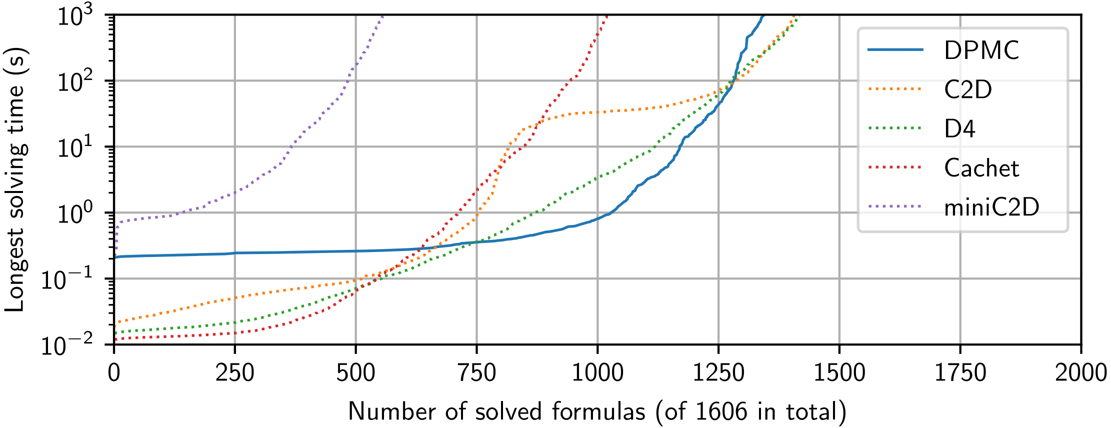
- 1606 CNF formulas:
  - 1049 benchmarks from Bayesian inference [SBK05]
  - 577 benchmarks from planning [PG09]
- State-of-the-art model counters:
  - `Cachet` [San+04]
  - `C2D` [Dar04]
  - `miniC2D` [OD15]
  - `D4` [LM17]
- NOTS cluster at Rice University:
  - CPU: single cores
  - RAM: 25 GB
  - Time: 1000 seconds per solver per benchmark

Cactus Plot: Planner-Executor Combinations



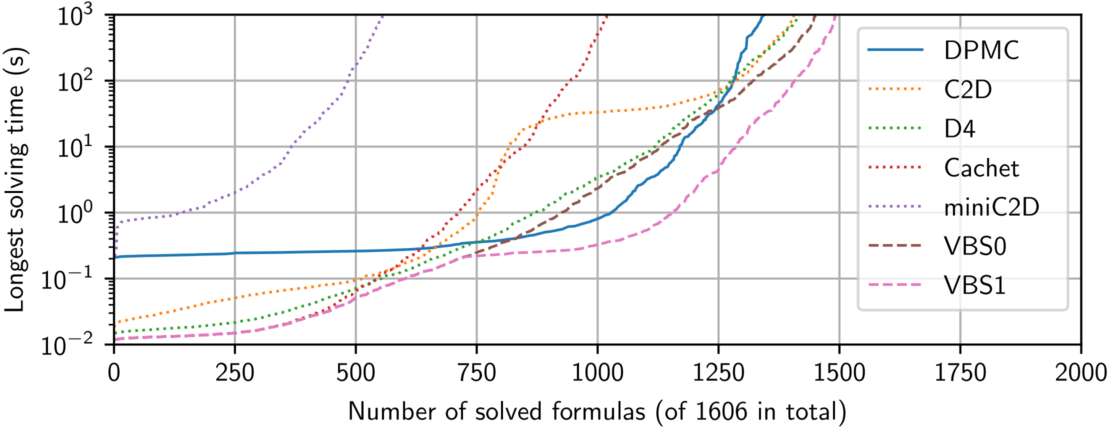
Tree-decomposition (TD) planner outperforms constraint-programming (CP) planner

Cactus Plot: Actual Solvers



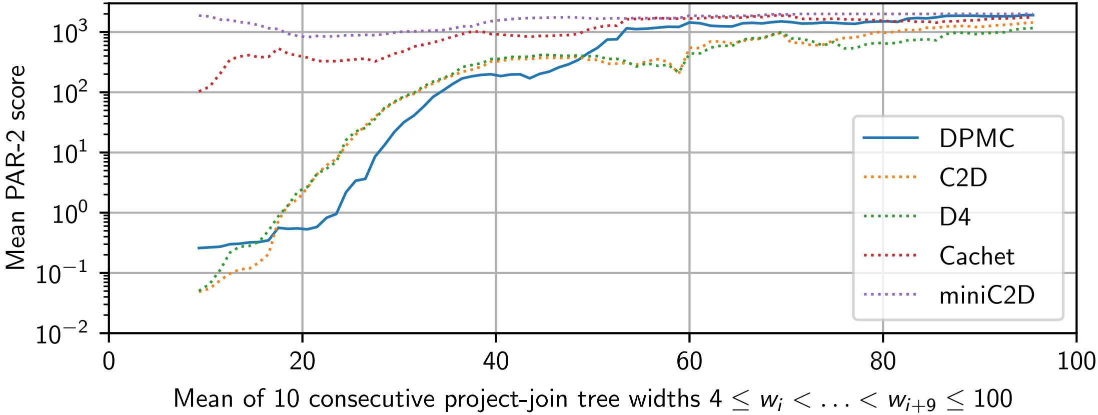
`DPMC` (TD planner and ADD executor) is competitive

Cactus Plot: Virtual Best Solvers



Virtual best solver `VBS1` (with `DPMC`) significantly outperforms `VBS0` (without `DPMC`)

Performance on 1522 Project-Join Trees for 1606 Benchmarks (94%)



$$\text{PAR2score}\,(\mathit{tool}, \varphi) := \begin{cases} \text{solving time} & \text{if } \mathit{tool} \text{ solves } \varphi \text{ within 1000 seconds} \\ 2000 & \text{otherwise} \end{cases}$$

Progress

Weighted SAT

- Applications of weighted SAT and its equivalent, MaxSAT:
  - Verification of neural networks [Sak20]
  - Synthesis of hardware exploits [Zha+20]
- Input 1: a Boolean formula φ, where $S = \text{vars}(\varphi)$
- Input 2: a weight function W mapping assignments on S to positive real-valued weights
- Output: a model of φ with the highest weight

$$\max_{S} ([\varphi] \cdot W) \qquad \text{the } \textit{maximum}$$

$$\operatorname*{argmax}_{S} ([\varphi] \cdot W) \qquad \text{a } \textit{maximizer}$$

Finding Maximizers

- Pseudo-Boolean function $f : \mathbb{B}^S \to \mathbb{R}$
- *Derivative sign* $\operatorname{dsgn}_x f : \mathbb{B}^{S \setminus \{x\}} \to \mathbb{B}^{\{x\}}$

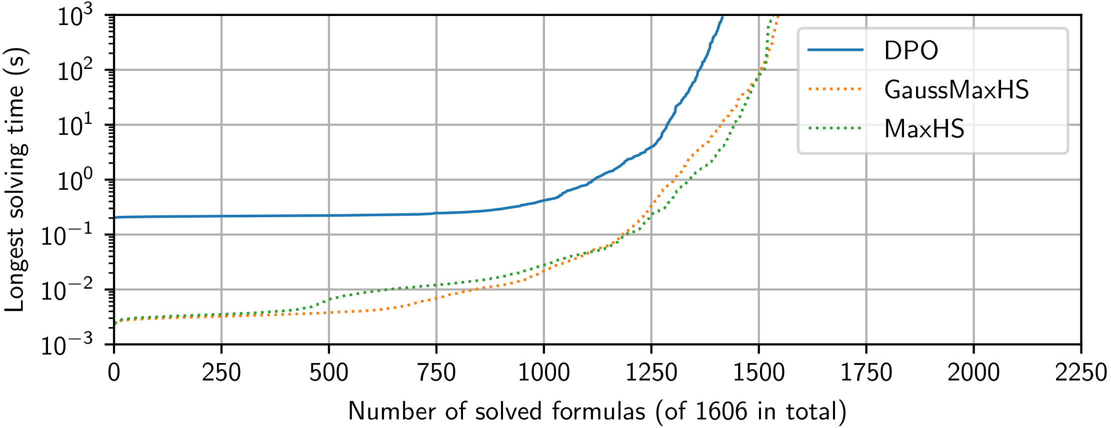
$$\left(\operatorname*{dsgn}_x f\right)(\tau) := \begin{cases} \{\langle x, 1\rangle\} & \text{if } f(\tau \cup \{\langle x, 1\rangle\}) \geq f(\tau \cup \{\langle x, 0\rangle\}) \\ \{\langle x, 0\rangle\} & \text{otherwise} \end{cases}$$

- *Iterative maximization* in pseudo-Boolean programming [CHJ90] and MaxSAT [KVZ22]:
  - Let $g = \max_x f : \mathbb{B}^{S \setminus \{x\}} \to \mathbb{R}$
  - Assume $\tau \in \mathbb{B}^{S \setminus \{x\}}$ is a maximizer of g
  - Then $\alpha = \tau \cup (\operatorname{dsgn}_x f)(\tau) \in \mathbb{B}^S$ is a maximizer of f
- `DPO` (dynamic-programming optimizer):
  - Use iterative maximization to find a maximizer
  - Adapt `DPMC` (model counting) to find the maximum

Evaluation of Weighted-SAT Solvers

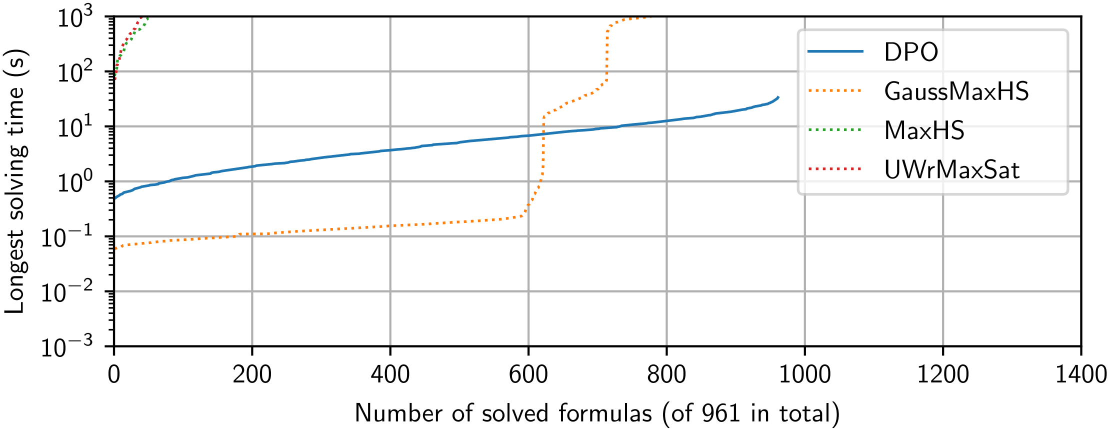
- Benchmarks:
  - 1606 application formulas in CNF from model counting
  - 961 crafted formulas in XOR-CNF from MaxSAT [KVZ22, Zhiwei]
- State-of-the-art solvers:
  - `UWrMaxSat` [Pio20]
  - `MaxHS` [DB11]
  - `GaussMaxHS` [SM21]

Application CNF Benchmarks



`UWrMaxSat` does not support floating-point weights

Crafted XOR-CNF Benchmarks



Neither `UWrMaxSat` nor `MaxHS` supports XOR

Progress

Projected Counting

- Applications of projected counting:
  - Planning [Azi+15]
  - Sampling [Gup+19]
- Input 1: a Boolean formula φ
- Input 2: a partition $\{X, Y\}$ of $\text{vars}(\varphi)$
  - X: *summative variables*
  - Y: *maximal variables*
- Output: the number of assignments τ on X such that $\varphi \mid \tau$ is satisfiable

$$\sum_X \max_Y [\varphi] \qquad \textit{projected count}$$

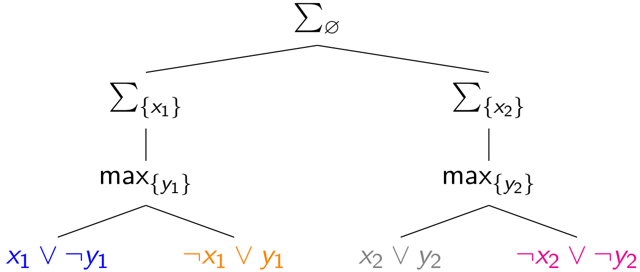
- Summative projection does not commute with maximal projection:

$$\sum_x \max_y f \neq \max_y \sum_x f \qquad \text{in general}$$

Graded Project-Join Tree

$$\sum_X \max_Y [(x_1 \vee \neg y_1) \wedge (\neg x_1 \vee y_1) \wedge (x_2 \vee y_2) \wedge (\neg x_2 \vee \neg y_2)]$$

projected count



$\langle X, Y \rangle$-*graded* project-join tree: X nodes are closer to the root than Y nodes

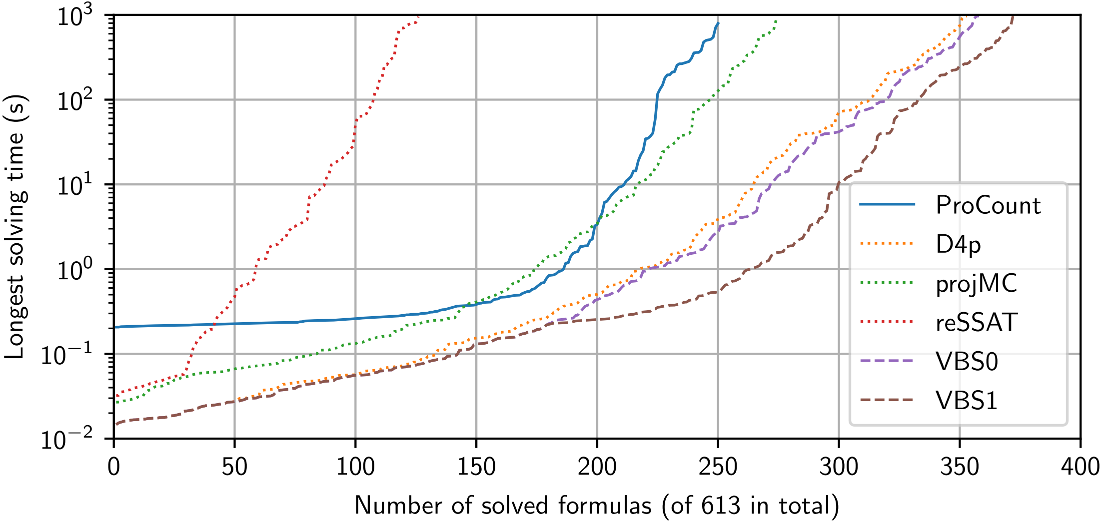
Adapting `DPMC` (Model Counting) for Projected Counting

- Reduction from graded project-join trees to ungraded project-join trees (Jeff)
- `ProCount` (projected counter):
  - Adapt planners:
    - tree decompositions (Jeff)
    - constraint programming
  - Adapt executor: ADDs

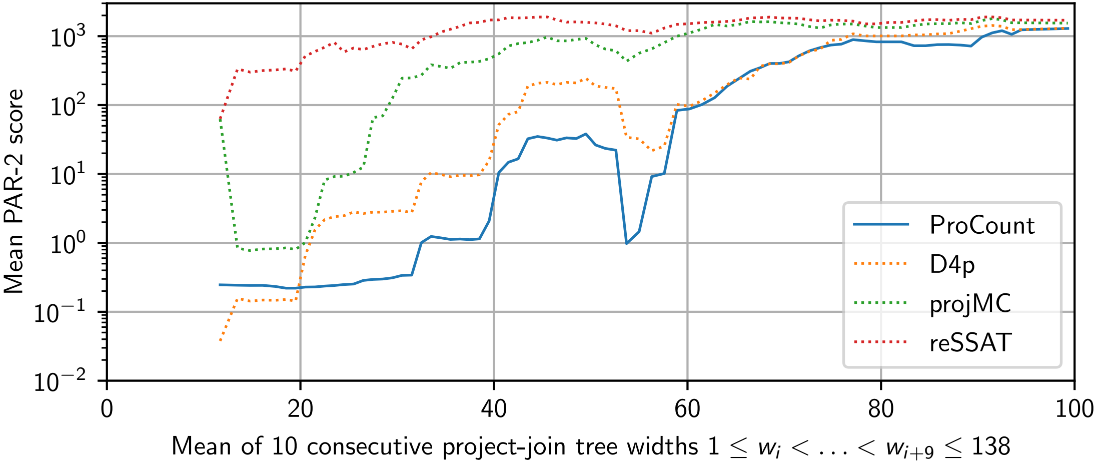
Evaluation of Projected Counters

- 613 CNF formulas:
  - 500 benchmarks from projected counting [SM19]
  - 113 benchmarks from projected sampling [Gup+19]
- State-of-the-art projected counters:
  - `reSSAT` [LWJ17]
  - `D4p` [LM17]
  - `projMC` [LM19]

Cactus Plot: Virtual Best Solvers



Performance on 291 Project-Join Trees for 613 Benchmarks (47%)



`ProCount` is fast on benchmarks whose project-join trees have low widths (below 60)

Progress

Exist-Random SAT (ERSAT)

- Applications of ERSAT:
  - Planning [ML98]
  - Fairness in machine learning [GBM21]
- Input 1: a Boolean formula φ
- Input 2: a partition $\{X, Y\}$ of $\text{vars}(\varphi)$
  - X: maximal variables
  - Y: summative variables
- Output: an assignment τ on X that maximizes the model count of $\varphi \mid \tau$

$$\max_X \sum_Y [\varphi] \qquad \text{the maximum}$$

$$\operatorname{argmax}_X \sum_Y [\varphi] \qquad \text{a maximizer}$$

- `DPER` (dynamic-programming ERSAT solver):
  - Adapt `ProCount` (projected counting) to find the maximum
  - Adapt `DPO` (weighted SAT) to find a maximizer

Evaluation of ERSAT Solvers

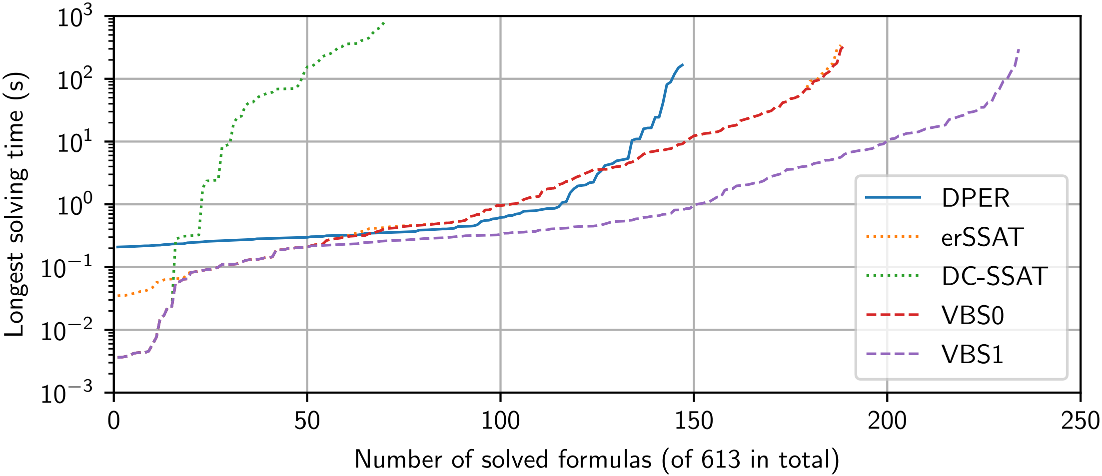
- 613 CNF formulas from projected counting:

$$\sum_{X} \max_{Y} f \qquad \text{original}$$

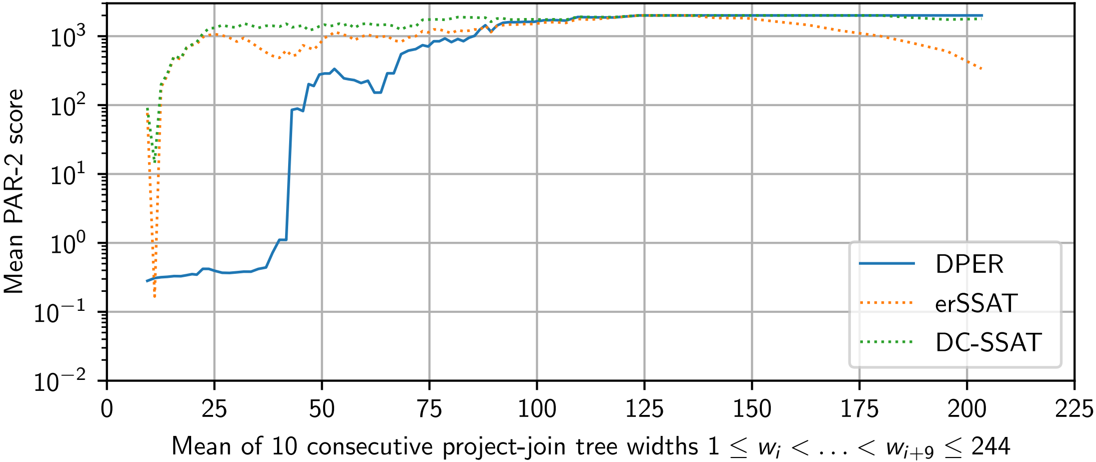
$$\max_{Y} \sum_{X} f \qquad \text{adapted for ERSAT}$$

- State-of-the-art ERSAT solvers:
  - `erSSAT` [LWJ18]
  - `DC-SSAT` [MB05]

Cactus Plot: Virtual Best Solvers

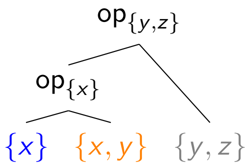


Performance on 204 Project-Join Trees for 613 Benchmarks (33%)



DPER is fast on benchmarks whose project-join trees have low widths (below 80)

Summary: Versatile Framework for Quantitative Reasoning



Project-join tree for $x \wedge (x \vee \neg y) \wedge (y \oplus z)$, where $\mathrm{op} := \begin{cases} \sum & \text{for summative variables} \\ \max & \text{for maximal variables} \end{cases}$

- Planning phase:
  - Tree decompositions (TDs)
  - Constraint programming
- Execution phase:
  - Tensors
  - Algebraic decision diagrams (ADDs)

Single plan with multiple executions:

- Model counting
- Weighted SAT
- Projected counting
- Exist-random SAT (ERSAT)

Unifying Current Work and Proposing Future Work

Current work: two projection operators and two join operators

Project-join tree	Projections	Join	Problem	Students
Ungraded	$\sum_S$	$\prod_{c \in \varphi} [c]$	Model counting [DPV20b]	Vu, Jeff
	$\max_S$	$\prod_{c \in \varphi} [c]$	Weighted SAT [PV22b]	Vu
	$\max_S$	$\sum_{c \in \varphi} [c]$	MaxSAT [KVZ22]	Zhiwei
Graded	$\sum_X \max_Y$	$\prod_{c \in \varphi} [c]$	Projected counting [DPV21]	Vu, Jeff
	$\max_X \sum_Y$	$\prod_{c \in \varphi} [c]$	ERSAT [PV22a]	Vu
	$\min_X \max_Y$	$\sum_{c \in \varphi} [c]$	*MinMaxSAT* [KVZ22]	Zhiwei

Future work:

- Hybrid inputs [KVZ22]:
  - Cardinality constraints
  - Pseudo-Boolean constraints
- Executors:
  - Multi-core ADDs [DP15]
  - Database engines [Fic+20]

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