ADDMC: Weighted Model Counting with Algebraic Decision Diagrams

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Problem: Boolean Model Counting

#SAT: computing number of satisfying assignments of Boolean formula

- Complexity: #P-complete [8]
- Example applications:
  - Medical diagnosis [7]
  - Power-transmission reliability analysis [3]

Weighted Model Counting

Boolean formula F and weight function W

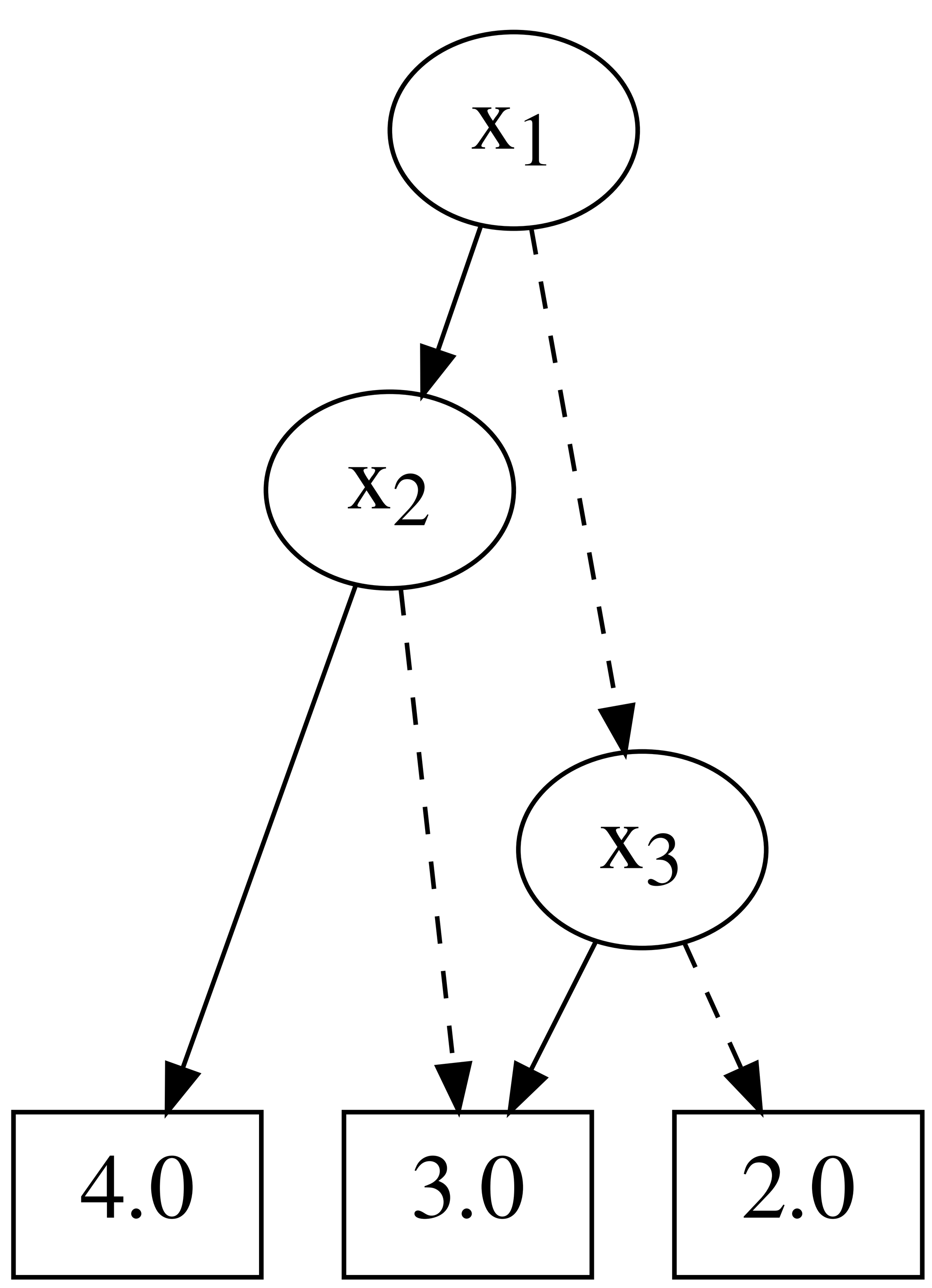
Assignment α			$F(\alpha)$	$W(\alpha)$	$(F \cdot W)(\alpha)$
x_1	x_2	x_3			
0	0	0	0	2.0	0.0
0	0	1	0	3.0	0.0
0	1	0	1	2.0	2.0
0	1	1	0	3.0	0.0
1	0	0	1	3.0	3.0
1	0	1	1	3.0	3.0
1	1	0	1	4.0	4.0
1	1	1	1	4.0	4.0

Weighted model count of F w.r.t. W

$$\#(F, W) = \sum_{\alpha \in 2^V} (F \cdot W)(\alpha) = 16.0$$

Algebraic Decision Diagram (ADD) [1]

Efficient data structure for weight function W[a]



[a] as well as for formula F and formula-weight product $F \cdot W$

Related Work on Model Counting

1. Search: exploring solution space in DPLL-like manner
   - `Cachet` [6]
2. Knowledge compilation: representing formula with efficient data structure *(exponential blowup in worst case)*
   - `c2d` [2]
   - `miniC2D` [5]
   - `d4` [4]

Contributions to Model Counting

1. Algorithm for exact weighted model counting
   - Using efficient data structure: ADD
   - Exploiting factored representation of Boolean formula
   - Employing dynamic programming to combine ADDs of factors of formula *(mitigating exponential blowup)*
2. Implementation: ADD model counter (`ADDMC`)
   - Source code and experimental data publicly available[a]

[a] https://github.com/vardigroup/ADDMC

Dynamic Programming

Factored Representation

Conjunctive Normal Form (CNF) Boolean formula

$$\varphi = (x_1 \lor x_3) \land (\neg x_2 \lor x_3) \land (x_2 \lor \neg x_3) \land x_3$$

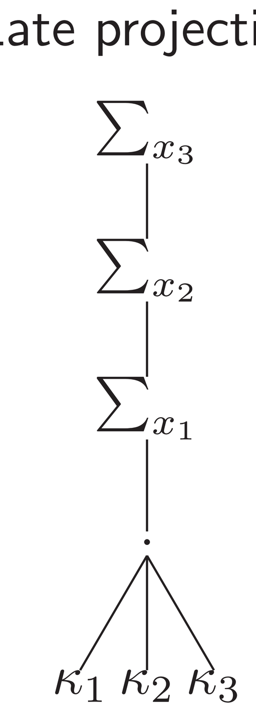
Clusters (partition of clauses)

$$\kappa_1 = \{x_1 \lor x_3\}$$
$$\kappa_2 = \{\neg x_2 \lor x_3, x_2 \lor \neg x_3\}$$
$$\kappa_3 = \{x_3\}$$

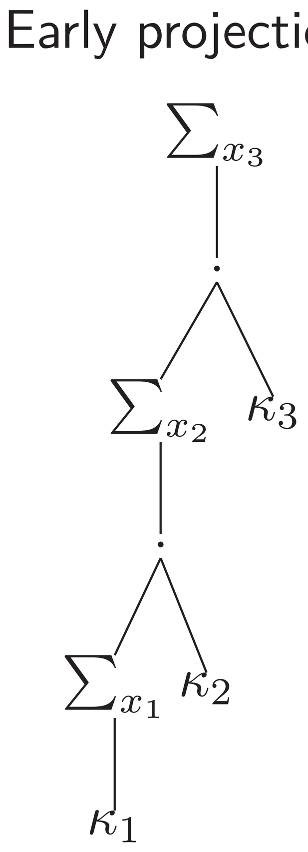
Early Projection

Reducing size of intermediate computation

Late projection



Early projection



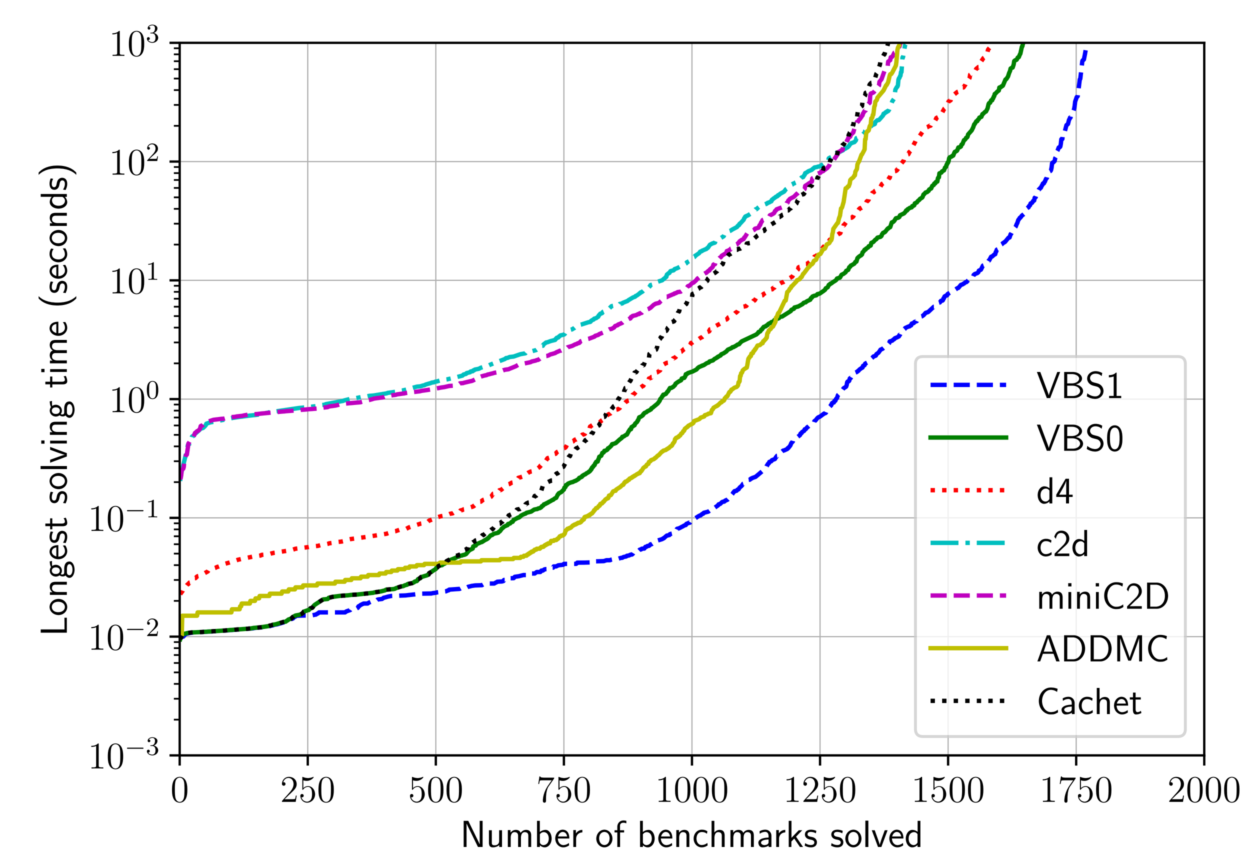
Experimental Benchmarks

1914 Boolean model counting benchmarks

- 1091 Bayesian networks[a]
- 823 instances from other domains[b]

[a] https://www.cs.rochester.edu/u/kautz/Cachet/
[b] http://www.cril.univ-artois.fr/KC/benchmarks.html

Experimental Results



Cactus plot of performance of two virtual best solvers (`VBS1` with `ADDMC`; `VBS0` without `ADDMC`) and five actual solvers

Acknowledgments

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References

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